Smoothness of Keller-Segel systems with critical diffusions

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I will present one of the most popular PDEs stemming from the mathematical biology, namely the Keller-Segel system.

The first part of my talk will present applicational motivation and classical rudiments of the analysis of the Keller-Segel system.

In the second part, I will focus on Keller-Segel systems with general diffusions, including semilinear and fractional ones. In particular, a disproof of the blowup conjecture for the critical, fractional one-dimensional case will be sketched. It is based on the ingenious methodology of moduli of continuity by Kiselev, Nazarov, Shterenberg and Volberg. If time permits, I will announce our most recent results on regularity in the supercritical regime. The second part of the talk will be based on a joint work with Rafael Granero-Belinchón (Davies) and Tomasz Cieślak (Warsaw).

Inviscid incompressible limits for rotating fluids

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We consider the inviscid limit for rotating compressible Navier-Stokes system. We show that the limit system is represented by the rotating incompressible Euler equation on the whole space. The rate of convergence is estimated in terms of the Mach and Reynolds numbers.

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Existence of the solution of degenerated PDEs in weighted Sobolev spaces with non-homogeneous boundary condition

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Singular quasilinear elliptic problems on unbounded domains

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We prove the existence of a solution between an ordered pair of sub and supersolutions for singular quasilinear elliptic problems on unbounded domains. Further, we use this result to establish the existence of a positive solution to the problem

$$\begin{cases} -\Delta_p u = \lambda K(x) f(u) & \text{in } B_1^c, \\ u = 0 & \text{on } \partial B_1, \\ u(x) \to 0 & \text{as } |x| \to \infty, \end{cases}$$

where $B_1^c = \{x \in \mathbb{R}^n \mid |x| > 1\}$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u), 1 < p < n$, λ is a positive parameter, K belongs to a class of functions which satisfy certain decay assumptions and f belongs to a class of $(p-1)-$subhomogeneous functions which may be singular at the origin, namely $\lim_{s\to 0^+} f(s) = -\infty$. Our methods can be also applied to establish a similar existence result when the domain is entire $\mathbb{R}^n$.

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Topological soliton models - electric charge as topological charge

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One of the basic properties of physics is charge quantization: requiring charge to be an integer multiplicity of elemental charge. Related not understood issue is infinite energy of electric field of point charge. I will introduce to topological models of particles, which among other allow to solve both issues: charge is quantized due to topology, singularity of energy is removed thanks to nonlinear Higgs-like potential. I will start with simple $1D$ sine-Gordon model which has charge quantization, special relativity effects (Lorentz contraction, time dilation, mass increase) and soliton mass (rest energy due to Higgs-like potential), released while annihilation. Higher dimensional models will additionally get long range attraction (repulsion) for opposite (the same) charges : $\frac{1}{r}$ in $2D$ and finally $\frac{1}{r^2}$ Coulomb interaction in $3D$.

Hardy inequality in variable exponent spaces derived from nonlinear problem

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We derive a family of weighted $p(x)$–Hardy inequalities with an additional term of the form

$$\int_\Omega |\xi|^{p(x)}\mu_1(dx) \leq \int_\Omega |\nabla\xi|^{p(x)}\mu_2(dx) + \int_\Omega |\xi\log\xi|^{p(x)}\,\mu_3(dx),$$

where $1 < p^- := \operatorname{ess\,inf}_{x\in\Omega} p(x) \leq p(x) \leq p^+ := \operatorname{ess\,sup}_{x\in\Omega} p(x) < \infty$, $p \in W^{1,1}_{loc}(\Omega)$, $p^{p(x)}, |\nabla p|^{p(x)} \in L^1_{loc}(\Omega)$, $\xi : \Omega \to \mathbb{R}$ is compactly supported Lipschitz function, and Ω is an open subset of $\mathbb{R}^n$, not necessarily bounded. The involved measures $\mu_1(dx), \mu_2(dx), \mu_3(dx)$ depend on $p(x)$, a certain parameter β, a piecewise continuous function $\sigma(x)$, and a nonnegative weak solution u to the PDI

$$-\Delta_{p(x)}u \geq \Phi \quad \text{in} \quad \Omega,$$

with a locally integrable function Φ. We admit the functions $\sigma(x)$ and Φ such that $\Phi \cdot u + \sigma(x)|\nabla u|^{p(x)} \geq 0$ a.e. in Ω. As a consequence of Caccioppoli–type inequality for the solution u we get Hardy inequality with an additional term in variable exponent Lebesgue spaces.

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Weighted iterated Hardy-type inequalities

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In this lecture we present a reduction and equivalence theorems for the boundedness of the composition of a quasilinear operator T with the Hardy and Copson operators in weighted Lebesgue spaces. New equivalence theorems are obtained for the operator T to be bounded in weighted Lebesgue spaces restricted to the cones of monotone functions, which allow to change the cone of non-decreasing functions to the cone of non-increasing functions and vice versa not changing the operator T. New characterizations of the weighted Hardy-type inequalities on the cones of monotone functions are given. The validity of so-called weighted iterated Hardy-type inequalities are characterized.

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On the convergence of double fourier series of Functions of bounded Partial Generalized Variation

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The convergence of double Fourier series of functions of bounded partial Λ -variation is investigated. The sufficient and neccessary conditions on the sequence $\Lambda = \{\lambda_n\}$ found for the convergence of partial sums of Fourier series of functions of bounded partial Λ -variation.

Symmetry-breaking bifurcation for free elastic shell of Biological Cluster

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We will be concerned with a two-dimensional mathematical model for a free elastic shell of biological cluster. The cluster boundary is connected with its kernel by elastic links. The inside part is filled by compressed gas or fluid. Equilibrium forms of the shell of biological cluster may be found as solutions of a certain nonlinear functional-differential equation with several physical parameters. For each multiparameter this equation has a radially symmetric solution. Our goal is to study the bifurcation which breaks symmetry. In order to establish critical values of bifurcation parameter and buckling modes we will investigate an appropriate linear problem. Our main result on the existence of symmetry-breaking bifurcation will be proved by the use of a variational version of the Crandall-Rabinowitz theorem.

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Sobolev homeomorphism that cannot be approximated by diffeomorphisms in $W^{1,1}$

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We construct a Sobolev homeomorphism in dimension $n \geq 4$, $f \in W^{1,1}((0,1)^n, \mathbb{R}^n)$ such that $J_f = \det Df > 0$ on a set of positive measure and $J_f < 0$ on a set of positive measure. It follows that there are no diffeomorphisms (or piecewise affine homeomorphisms) f_k such that $f_k \to f$ in $W^{1,1}_{loc}$. This is a joint result with B. Vejnar.

Nodal and multiple solutions for a nonhomogeneous Neumann boundary problem

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We consider a nonlinear Neumann problem driven by a p-Laplacian-type, nonhomogeneous elliptic differential operator and a Carathéodory reaction term with zeroes. We prove the existence of two extremal constant sign smooth solutions and a nontrivial nodal smooth solution. In the proof we use variational methods with truncation techniques, critical point theory and Morse theory (critical groups).

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Models of fatigue accumulation in elasto-plastic materials

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We propose a model for fatigue accumulation based on the hypothesis that there exists a proportionality between fatigue and dissipated energy. We demonstrate our model on the example of a transversally oscillating elastoplastic beam. The full system consists of the momentum and energy balance equations, and an evolution equation for the fatigue rate. In nontrivial cases, the process develops a singularity (material failure) in finite time. The main result consists in proving the existence and uniqueness of a strong solution in a time interval depending on the size of the data.

We will also discuss a models for fatigue accumulation in an oscillating elastoplastic beam under the additional hypothesis that the material can partially recover by the effect of melting.

This is a joint work with Pavel Krejčí and Michela Eleuteri.

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First order systems of ODEs with nonlinear nonlocal boundary conditions

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We consider the following ordinary differential equation (1):

$$x' = f(t, x) \tag{0.1}$$

with the non-local condition (2):

$$h\left(\int_0^1 x(s)\, dg(s)\right) = 0, \tag{0.2}$$

where $f : [0,1] \times \mathbb{R}^k \to \mathbb{R}^k$ is continuous, $g = (g^1, \dots, g^k) : [0,1] \to \mathbb{R}^k$ has bounded variation, $h : \mathbb{R}^k \to \mathbb{R}^k$ is continuous and

$$\int_0^1 x(s)\, dg(s) = \left(\int_0^1 x^1(s)\, dg^1(s), \dots, \int_0^1 x^k(s)\, dg^k(s)\right).$$

We are intrested in the BVP for equation (1) with (2). The basic idea is to apply the problem (1), (2) as abstract nonlinear equation

$$Lx = N(x). \tag{0.3}$$

We shall use the Mawhin's coincidence theory to prove existence of solutions (1), (2).

The same BVP will be considered in the case where the function f has singularities at the ends $[0,1]$.

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The three critical point result for eigenvalue problem with perturbations for Laplace operator

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On the condition of tetrahedral polyconvexity, arising from calculus of variations

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Homoclinic orbits for an almost periodically forced singular Newtonian system in $\mathbb{R}^3$

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We will be concerned with the existence of homoclinic solutions for a Newtonian system $\ddot{q} + a(t)\nabla W(q) = 0$, where $t \in \mathbb{R}$, $q \in \mathbb{R}^3$. It is assumed that there is a line $l \in \mathbb{R}^3 \setminus \{0\}$ such that a potenial $W \in C^2(\mathbb{R}^3 \setminus \{l\}, \mathbb{R})$ has a global maximum at the origin and the line l consists of singular points. Moreover, W satisfies the "strong-force" condition in a neighbourhood of l and $a : \mathbb{R} \to \mathbb{R}$ is a continuous almost periodic function. The existence of at least two solutions will be discussed.

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Unsaturated fluid flow interacting with a deformable porous medium

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Unsaturated porous media consist of three components: A solid skeleton, a liquid, and a gas. The pressures at the interfaces of the two pore fluids are not the same. The difference is called the capillary pressure. In a soil filled by a water-air mixture, the value of the capillary pressure is higher if an initially water-saturated soil is drained than if an initially air-filled soil is irrigated. Thus, the constitutive pressure-saturation relation depends also on the history of the process, and a phenomenon called "soil-moisture hysteresis" occurs. We refer to the work of Flynn [2] in which various Preisach hysteresis operators are investigated and their parameters are fitted to drying, wetting and scanning curves of several soil types.

We derive our model from the first and the second principle of thermodynamics in Lagrange coordinates, taking into account hysteresis both in the elastoplastic constitutive law for a deformable solid matrix, and in the pressure-saturation relation for the liquid. In the isothermal case, the resulting system of PDEs with hysteresis reads as follows:

$$\begin{aligned} \rho_S u_{tt} &= \operatorname{div}\left(\mathbf{B}\nabla_s u_t + \mathcal{P}[\nabla_s u]\right) + \nabla p + g\,, \\ \mathcal{G}[p]_t &= \operatorname{div} u_t + \frac{1}{\rho_L}\operatorname{div}\left(\mu(p)\nabla p\right), \end{aligned}$$

with unknowns u (displacement) and p (capillary pressure), where ρ_S, ρ_L are the mass densities of the solid and liquid, respectively, $\mathbf{B}$ is a constant viscosity matrix, $\mathcal{P}$ is the constitutive operator of elastoplasticity, ∇_s is the symmetric gradient, $\mathcal{G}$ is a Preisach operator describing the pressure-saturation hysteresis, $\mu(p)$ is the permeability coefficient, g is an external volume force. The system is complemented with suitable boundary and initial conditions. It is degenerate due to the fact that the values of the saturation $\mathcal{G}[p]$ are restricted to the interval $[0, 1]$. A detailed derivation of the model and the proof of existence of a global strong solution are given in [1].

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Singularities in ternary diffusion models

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A ternary system in Darken's setting is governed by a system of PDE's. The well-posedness and numerical approximations have been studied by M. Danielewski, K. Holly, H. Leszczyski, M. Wrzosek, M. Matusik. Both issues face huge difficulties and challenges: (i) how to prove that solutions are admissible and remain in a subset where the differential problem is parabolic? (ii) how to construct numerical schemes which are stable in some Sobolev space? (ii) are phenomena of instabilities caused by singularities? We apply a perturbation method which leads to a system of parabolic equations with possible degenerations. Our approach leads to new numerical schemes.

A very singular equation

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We will discuss the behaviour of solutions to a model strictly parabolic very singular equation

$$u_t = u_{xx} + (\mathrm{sgn} u_x)_x.$$

Their evolution turns out to reduce to a certain Stefan-like system of free boundary problems for heat equation.

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Attractors in some problems of fluid mechanics

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We consider the problem of existence and finite dimensionality of attractors for some classes of two-dimensional turbulent boundary driven flows which naturally appear in lubrication theory. The flows admit mixed, nonstandard boundary conditions and time dependent driving forces. In particular, we are interested in dependence of dimension of the attractors on geometry of the flow domain and on the boundary conditions.

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Singularities of biharmonic maps

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The purpose of this talk is to review some known results and work in progress concerning singularities of the biharmonic maps, i.e. critical points of the Hessian energy $\int_\Omega |\Delta u|^2 dx$ among maps $u \in W^{2,2}(\Omega, \mathbb{S}^k)$.

Time-dependent Navier-Stokes equations with deterministic and random forces

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We prove the existence of a martingale solution of the stochastic Navier-Stokes equations in 2D and 3D domains. Using the Yamada-Watanabe approach, we prove also the existence and uniqueness of strong solutions in the case of 2D domain. The presentation is based on the joint work with Z. Brzeźniak.

Optimality of Orlicz spaces in Sobolev embeddings

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We prove the existence of a martingale solution of the stochastic Navier-Stokes equations in 2D and 3D domains. Using the Yamada-Watanabe approach, we prove also the existence and uniqueness of strong solutions in the case of 2D domain. The presentation is based on the joint work with Z. Brzeźniak.

Diffusion and low Mach number limits in a model of radiative flow.

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We consider relativistic and "semi-relativistic" models of radiative viscous compressible Navier-Stokes-Fourier system coupled to the radiative transfer equation extending the classical model introduced in [1] and we study some of its singular limits (low Mach and diffusion) in the case of well-prepared initial data and Dirichlet boundary condition for the velocity field. In the low Mach number case we prove the convergence toward the incompressible Navier-Stokes system coupled to a system of two stationary transport equations. In the diffusion case we prove the convergence toward the compressible Navier-Stokes with modified state functions (equilibrium case) or toward the compressible Navier-Stokes coupled to a diffusion equation (non equilibrium case).

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Penalization method for a class of variational-hemivariational inequalities with history operator

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We consider a class of variational-hemivariational inequalities with history operator and we prove an existence and uniqueness of a solution for such inequalities. Next, we use the penalized method in the study of variational-hemivariational inequalities with history operator. We prove the unique solvability of the penalized problems and the convergence of their solutions to the solution of the original problem, as the penalization parameter converges to zero.

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Embeddings of Sobolev-type spaces into generalized Hölder spaces involving k-modulus of smoothness

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We use an estimate of the k-modulus of smoothness of a function f such that the norm of its distributional gradient $|\nabla^k f|$ belongs locally to the Lorentz space $L^{n/k,1}(\mathbb{R}^n)$, $k \in \mathbb{N}$, $k \le n$, and its reverse form to establish necessary and sufficient conditions for continuous embeddings of Sobolev-type spaces. These spaces are modelled upon rearrangement invariant Banach function spaces $X(\mathbb{R}^n)$. Target spaces of our embeddings are generalized Hölder spaces defined by means of the k-modulus of smoothness ($k \in \mathbb{N}$). General results are illustrated with examples. Particular cases of these examples improve and extend known results.

The lecture is based joint works with Amiran Gogatishvili, Susana Moura and Júlio Neves.

Positive solutions of elliptic problems with a singular nonlinearity

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We investigate the existence and properties of solutions of the following elliptic differential equations

$$\operatorname{div}\left(a(||x||)\nabla u(x)\right) + f(x, u(x)) - (u(x))^{-\alpha}||\nabla u(x)||^{\beta} + g(||x||)x \cdot \nabla u(x)) = 0,$$

for $x \in \mathbb{R}^n$, $||x|| > R$. We discuss positive solutions such that $\lim\limits_{||x||\to\infty} u(x) = 0$. Our approach is based on the subsolution and supersolution method. Results described in [1] allowed us to obtain the existence of a positive evanescent supersolution of our problem.

References

[1] A.Orpel, Increasing sequences of positive evanescent solutions of nonlinear elliptic equations, *J. Differential Equations*, http://dx.doi.org/10.1016/j.jde.2015.03.010.

Interaction of particles through a singular potential

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Mathematical description of the collective dynamics of interacting particles finds many applications ranging from the modeling of flocks of birds, optimal control over sensor networks to some seemingly unrelated subjects as the modeling of a consensus in societies. One of such models is Cucker-Smale's (C-S) flocking model associated with the Vlasov-type equation

$$\partial_t f + v \cdot \nabla f + \operatorname{div}_v(F(f)f) = 0,$$

$$F(f)(x,v,t) := \int_{\mathbb{R}^{2d}} \psi(|x-y|)(w-v)f(y,w,t)dydw,$$

where ψ is a given function called *the communication weight*. In the case of regular ψ, C-S model was thoroughly studied and it's properties are well known (see e.g. [1] or [2]). However, with a singular ψ of the form $\psi(s) = s^{-\alpha}$ for $\alpha > 0$, it becomes much more difficult. Depending on α the trajectories of the particles may exhibit tendencies to behave in various ways: stick together, collide or avoid each other altogether (see [2, 3, 4, 5]). Though much effort was put into the study of dynamics of C-S model with a singular weight, yet the existence of the solutions is still not known in most cases. In my talk I will present the proof of existence of solutions to the C-S model provided that $0 < \alpha < \frac{1}{2}$ (see [6]). I will also show some examples of it's dynamics.

References

[1] Ahn S. M. and Choi H. and Ha S.-Y. and Lee H.,*On collision-avoiding initial configurations to Cucker-Smale type flocking models*, Commun. Math. Sci., **10**, No. 2, 625–643 (2012)

[2] Carrillo J., A. and Choi Y.-P. and Hauray M.,*Local well-posedness of the generalized Cucker-Smale model*, preprint, arXiv:1406.1792 (2014)

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[6] Peszek J.,*Discrete Cucker-Smale's flocking model with a weakly singular weight*, submited, arXiv:1412.6458v1 (2014)

Marcinkiewicz theorems for Orlicz spaces (a tribute to great Polish mathematicians)

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Assuming that a linear operator is bounded between two pairs of Marcinkiewicz spaces, we ask when it is bounded on a pair of intermediate Orlicz spaces. The investigation of this problem leads us first to proving an assertion in the spirit of the classical result of Dmitriev and Krein, and then to having to grapple various funny intrinsic properties of operators involving suprema.

References

[1] V.I. Dmitriev and S.G. Krein, *Interpolation of operators of weak type*, Anal. Math., **4(2)**(1978), 83-99.

Typicality of $J_f = 0$ almost everywhere for homeomorphisms in the plane

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Let us consider the space of Sobolev homeomorphisms, $f \in W^{1,p}([0,1]^2, [0,1]^2)$, f is identity on the boundary of unit square, for $p \in (1,2)$. We prove that the property $J_f = 0$ almost everywhere is the typical property in this space.

References

[1] Černý R., Roskovec T., *Typicality of $J_f = 0$ almost everywhere for homeomorphisms in the plane*, in preparation.

Error bounds on numerical solutions of the van Roosbroeck equations

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We present Composite Discontinuous Galerkin Method discretization of the stationary van Roosbroeck equations, which may be used for numerical modelling of semiconductor devices. In this presentation, we show results for the equilibrium state in $\mathbb{R}$ and $\mathbb{R}^2$. We demonstrate existence and error bounds of the discrete solutions. We also verify these theoretical results against numerical simulations.

Existence and uniqueness of a weak solution to the Nernst-Planck-Poisson system

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We will study the one-dimensional Nernst-Planck-Poisson (NPP) system of partial differential equations with the initial condition and nonlinear boundary condition which covers especially the full Chang-Jaffé (CJ) condition. The system is composed of a finite number of parabolic and one elliptic equations. Such a system describes many important physical and biological processes for example ionic diffusion in porous media, electrochemical and biological membranes as well as electrons and holes transport in semiconductors. By the boundary condition considered the physical system can be not only closed but also open. We will present a theorem on the existence and uniqueness of a weak solution and numerical simulations.

Almost-compact embeddings between Banach function spaces

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An embedding between two Banach function spaces over a nonatomic measure space is never compact. Therefore, it is natural to ask whether there is a suitable substitute for compactness in this situation. A positive answer to this question is provided by the notion of an almost-compact embedding. Although this embedding cannot be compact, it leads to compactness when combined with a continuous Sobolev embedding. In the talk we recall the definition of an almost-compact embedding and then present several characterizations and examples of such embeddings.

Simulation and numerical approach for some dynamical models in contact mechanics

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Embeddings between classical Lorentz spaces of type gamma defined with respect to two possibly different weighted integral means

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Perron's method for nonlocal boundary value problems in the context of viscosity solutions

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